

Symmetry

Chapter 13

13.1 Introduction

Symmetry is quite a common term used in day to day life. When we see certain figures with evenly balanced proportions, we say, “**They are symmetrical**”.



Tajmahal (U.P.)



Thiruvannamalai (Tamil Nadu)

These pictures of architectural marvel are beautiful because of their symmetry.

Suppose we could fold a picture in half such that the left and right halves match exactly then the picture is said to have line symmetry (Fig 13.1). We can see that the two halves are mirror images of each other. If we place a mirror on the fold then the image of one side of the picture will fall exactly on the other side of the picture. When it happens, the fold, which is the mirror line, is a line of symmetry (or an axis of symmetry) for the picture.

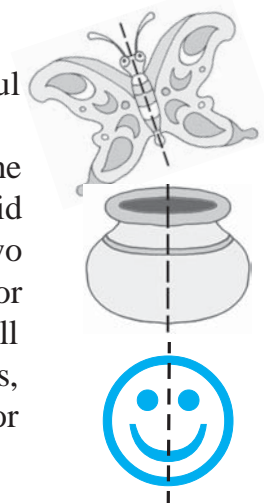


Fig 13.1

The shapes you see here are symmetrical. Why?
 When you fold them along the dotted line, one half of the drawing would fit exactly over the other half.
 How do you name the dotted line in the figure?
 Where will you place the mirror for having the image exactly over the other half of the picture?
 The adjacent figure 13.2 is not symmetrical.
 Can you tell 'why not'?

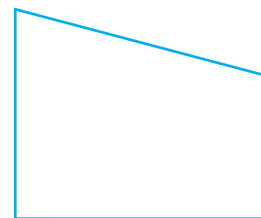


Fig 13.2

13.2 Making Symmetric Figures : Ink-blot Devils

Do This

Take a piece of paper. Fold it in half.

Spill a few drops of ink on one half side.

Now press the halves together.

What do you see?

Is the resulting figure symmetric? If yes, where is the line of symmetry? Is there any other line along which it can be folded to produce two identical parts?

Try more such patterns.



Inked-string patterns



Fold a paper in half. On one half-portion, arrange short lengths of string dipped in a variety of coloured inks or paints. Now press the two halves. Study the figure you obtain. Is it symmetric? In how many ways can it be folded to produce two identical halves?

Try These

You have two set-squares in your 'mathematical instruments box'. Are they symmetric?

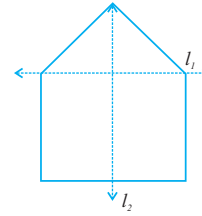
List a few objects you find in your class room such as the black board, the table, the wall, the textbook, etc. Which of them are symmetric and which are not? Can you identify the lines of symmetry for those objects which are symmetric?





EXERCISE 13.1

- List any four symmetrical objects from your home or school.
- For the given figure, which one is the mirror line, l_1 or l_2 ?
- Identify the shapes given below. Check whether they are symmetric or not. Draw the line of symmetry as well.

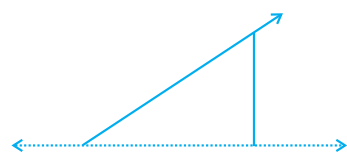


(a)	(b)	(c)
(d)	(e)	(f)

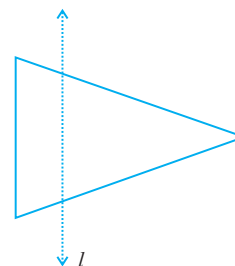
- Copy the following on a squared paper. A square paper is what you would have used in your arithmetic notebook in earlier classes. Then complete them such that the dotted line is the line of symmetry.

(a)	(b)	(c)
(d)	(e)	(f)

- In the figure, l is the line of symmetry. Complete the diagram to make it symmetric.



6. In the figure, l is the line of symmetry.
Draw the image of the triangle and complete the diagram so that it becomes symmetric.



13.3 Figures with Two Lines of Symmetry

Do This

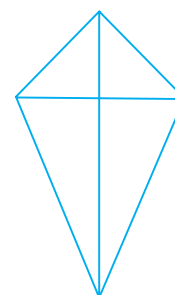
A kite

One of the two set-squares in your instrument box has angles of measure 30° , 60° , 90° .

Take two such identical set-squares. Place them side by side to form a 'kite', like the one shown here.

How many lines of symmetry does the shape have?

Do you think that some shapes may have more than one line of symmetry?

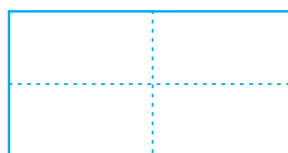


A rectangle

Take a rectangular sheet (like a post-card). Fold it once lengthwise so that one half fits exactly over the other half. Is this fold a line of symmetry? Why?



1st fold



2nd fold

Open it up now and again fold on its width in the same way. Is this second fold also a line of symmetry? Why?

Try These

Form as many shapes as you can by combining two or more set squares. Draw them on squared paper and note their lines of symmetry.

Do you find that these two lines are the lines of symmetry?

A cut out from double fold

Take a rectangular piece of paper. Fold it once and then once more. Draw some design as shown. Cut the shape drawn and unfold the shape. (Before unfolding, try to guess the shape you are likely to get).

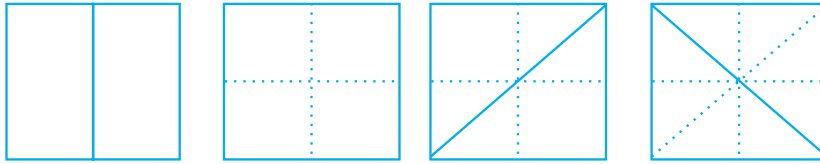


How many lines of symmetry does the shape have which has been cut out?

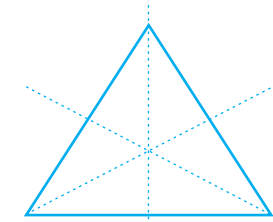
Create more such designs.



13.4 Figures with Multiple (more than two) Lines of Symmetry



Take a square piece of paper. Fold it into half vertically, fold it again into half horizontally. (i.e. you have folded it twice). Now open out the folds and again fold the square into half (for a third time now), but this time along a diagonal, as shown in the figure. Again open it and fold it into half (for the fourth time), but this time along the other diagonal, as shown in the figure. Open out the fold.



3 lines of symmetry for an equilateral triangle

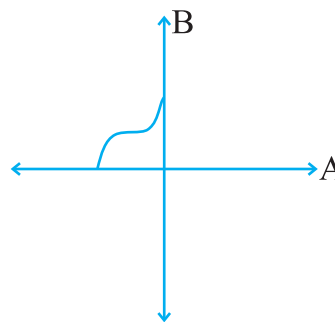
How many lines of symmetry does the shape have?

We can also learn to construct figures with two lines of symmetry starting from a small part as you did in Exercise 13.1, question 4, for figures with one line of symmetry.

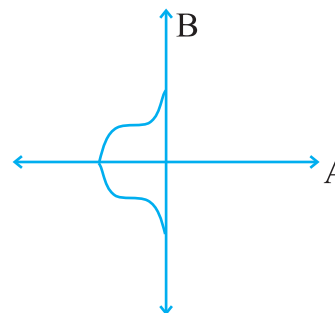
1. Let us have a figure as shown alongside.



2. We want to complete it so that we get a figure with two lines of symmetry. Let the two lines of symmetry be A and B.

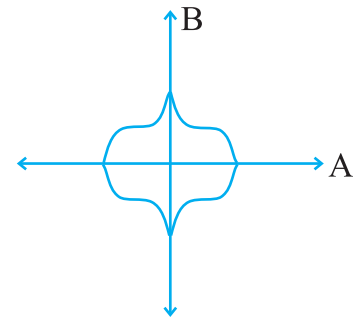


3. We draw the part as shown to get a figure having line A as a line of symmetry.



4. To complete the figure we need it to be symmetrical about line B also. Draw the remaining part of figure as shown.

This figure has two lines of symmetry i.e. line A and line B.



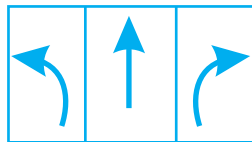
Try taking similar pieces and adding to them so that the figure has two lines of symmetry.

Some shapes have only one line of symmetry; some have two lines of symmetry; and some have three or more.

Can you think of a figure that has six lines of symmetry?

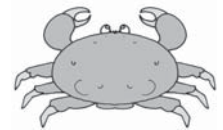
Symmetry, symmetry everywhere!

- Many road signs you see everyday have lines of symmetry. Here, are a few.

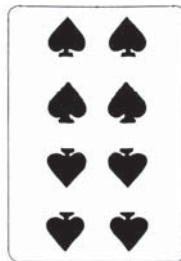
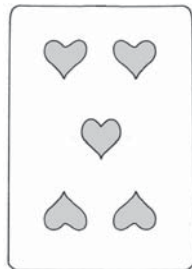


Identify a few more symmetric road signs and draw them. Do not forget to mark the lines of symmetry.

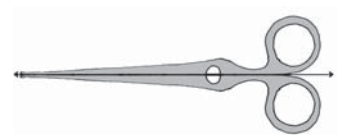
- The nature has plenty of things having symmetry in their shapes; look at these:



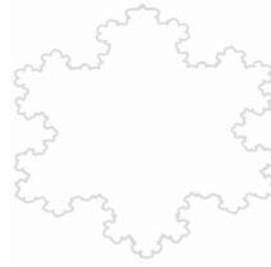
- The designs on some playing cards have line symmetry. Identify them for the following cards.



- Here is a pair of scissors!
How many lines of symmetry does it have?

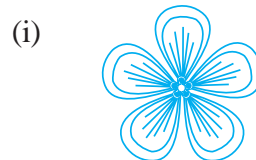
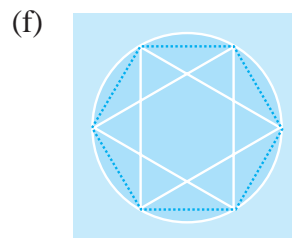
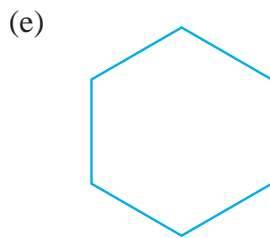
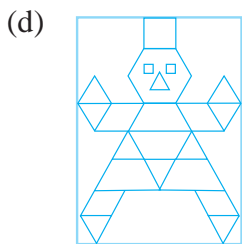
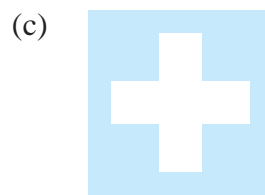
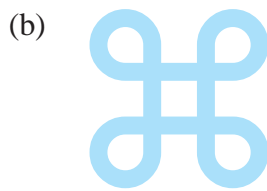
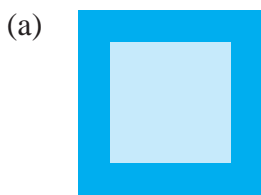


- Observe this beautiful figure.
It is a symmetric pattern known as Koch's Snowflake. (If you have access to a computer, browse through the topic "Fractals" and find more such beauties!). Find the lines of symmetry in this figure.

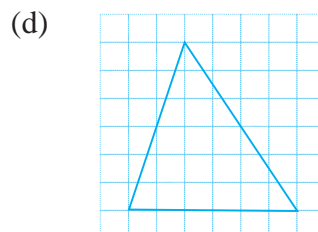
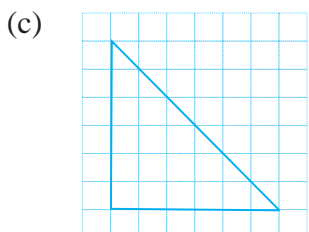
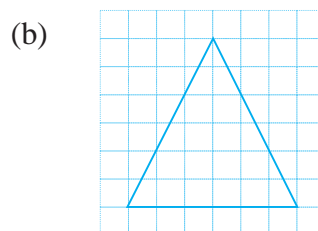
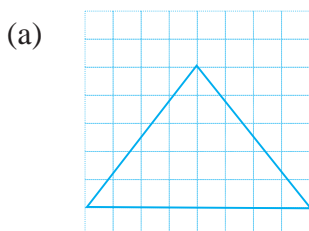


EXERCISE 13.2

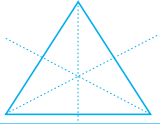
1. Find the number of lines of symmetry for each of the following shapes :



2. Copy the triangle in each of the following figures on squared paper. In each case, draw the line(s) of symmetry, if any and identify the type of triangle. (Some of you may like to trace the figures and try paper-folding first!)



3. Complete the following table.

Shape	Rough figure	Number of lines of symmetry
Equilateral triangle		3
Square		
Rectangle		
Isosceles triangle		
Rhombus		
Circle		

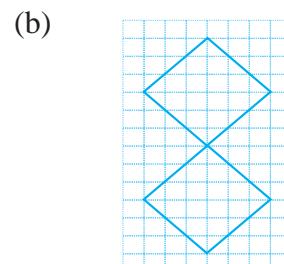
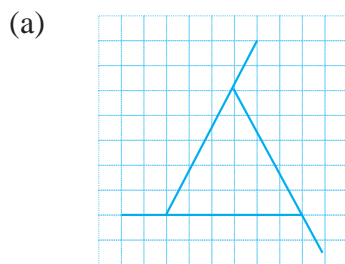
4. Can you draw a triangle which has
- exactly one line of symmetry?
 - exactly two lines of symmetry?
 - exactly three lines of symmetry?
 - no lines of symmetry?

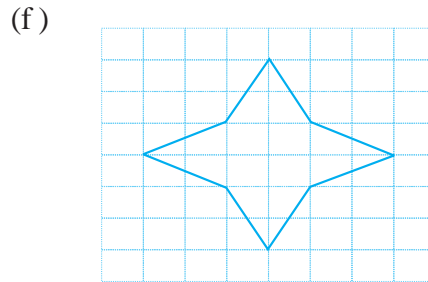
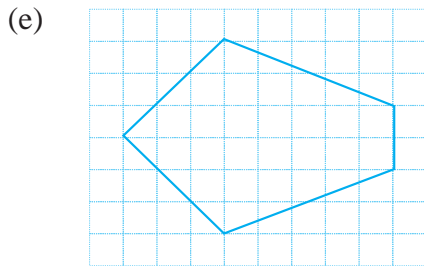
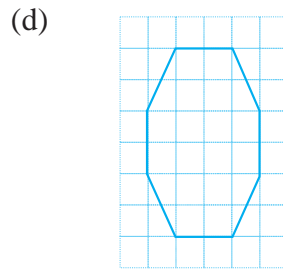
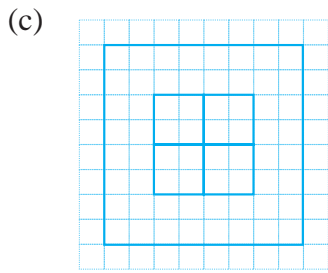
Sketch a rough figure in each case.

5. On a squared paper, sketch the following:
- A triangle with a horizontal line of symmetry but no vertical line of symmetry.
 - A quadrilateral with both horizontal and vertical lines of symmetry.
 - A quadrilateral with a horizontal line of symmetry but no vertical line of symmetry.
 - A hexagon with exactly two lines of symmetry.
 - A hexagon with six lines of symmetry.

(Hint : It will be helpful if you first draw the lines of symmetry and then complete the figures.)

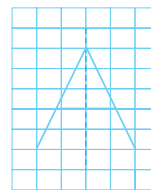
6. Trace each figure and draw the lines of symmetry, if any:



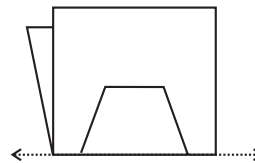
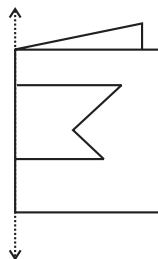


7. Consider the letters of English alphabets, A to Z. List among them the letters which have

- (a) vertical lines of symmetry (like A)
- (b) horizontal lines of symmetry (like B)
- (c) no lines of symmetry (like Q)



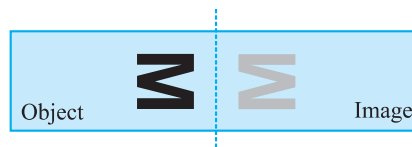
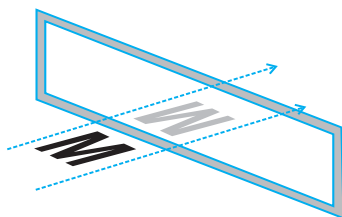
8. Given here are figures of a few folded sheets and designs drawn about the fold. In each case, draw a rough diagram of the complete figure that would be seen when the design is cut off.



13.5 Reflection and Symmetry

Line symmetry and mirror reflection are naturally related and linked to each other.

Here is a picture showing the reflection of the English letter M. You can imagine that the mirror is invisible and can just see the letter M and its image.



The object and its image are symmetrical with reference to the mirror line. If the paper is folded, the mirror line becomes the line of symmetry. We then say that the image is the reflection of the object in the mirror line. You can also see that when an object is reflected, there is no change in the lengths and angles; i.e. the lengths and angles of the object and the corresponding lengths and angles of the image are the same. However, in one aspect there is a change, i.e. there is a difference between the object and the image. Can you guess what the difference is?



(**Hint** : Look yourself into a mirror).

Do This



On a squared sheet, draw the figure ABC and find its mirror image A'B'C' with l as the mirror line.

Compare the lengths of AB and A'B'; BC and B'C'; AC and A'C'.

Are they different?

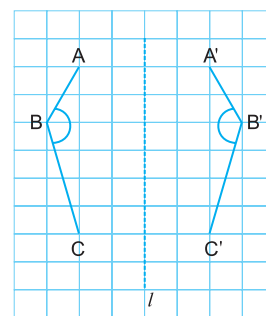
Does reflection change length of a line segment?

Compare the measures of the angles (use protractor to measure) $\angle ABC$ and $\angle A'B'C'$.

Does reflection change the size of an angle?

Join AA', BB' and CC'. Use your protractor to measure the angles between the lines l and AA', l and BB', l and CC'.

What do you conclude about the angle between the mirror line l and the line segment joining a point and its reflected image?



Try These

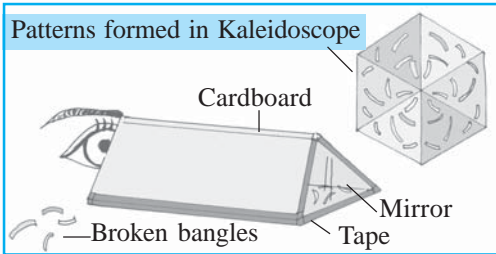
If you are 100 cm in front of a mirror, where does your image appear to be? If you move towards the mirror, how does your image move?

Paper decoration

Use thin rectangular coloured paper. Fold it several times and create some intricate patterns by cutting the paper, like the one shown here. Identify the line symmetries in the repeating design. Use such decorative paper cut-outs for festive occasions.



Kaleidoscope

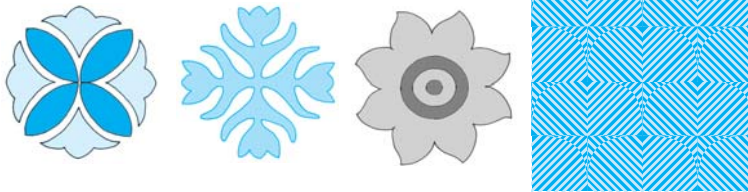


A kaleidoscope uses mirrors to produce images that have several lines of symmetry (as shown here for example). Usually, two mirror strips forming a V-shape are used. The angle between the mirrors determines the number of lines of symmetry.

Make a kaleidoscope and try to learn more about the symmetric images produced.

Album

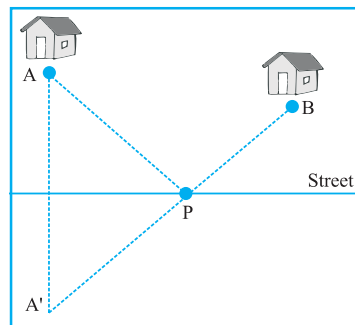
Collect symmetrical designs you come across and prepare an album. Here are a few samples.



An application of reflectional symmetry

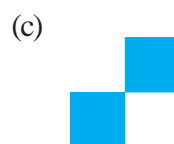
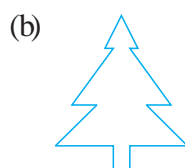
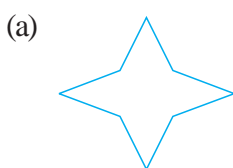
A paper-delivery boy wants to park his cycle at some point P and deliver the newspapers to houses A and B. Where should he park the cycle so that his walking distance $AP + BP$ will be least?

You can use reflectional symmetry here. Let A' be the image of A in the mirror line which is the street here. Then the point P is the ideal place to park the cycle (where the mirror line and $A'B$ meet). Can you say why?



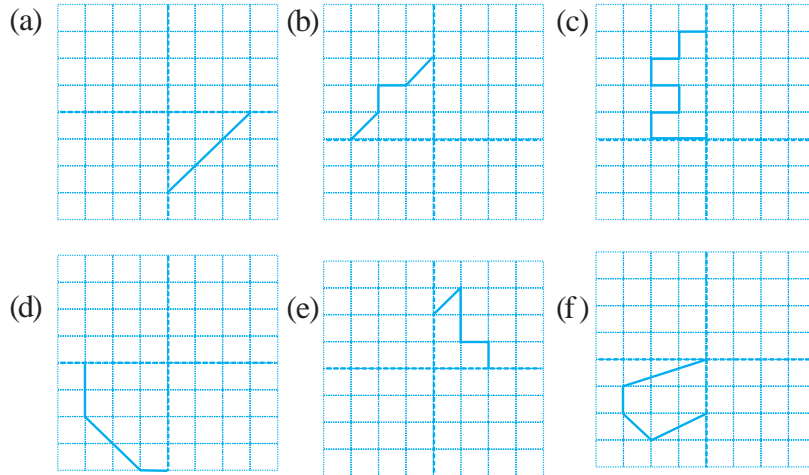
EXERCISE 13.3

- Find the number of lines of symmetry in each of the following shapes. How will you check your answers?



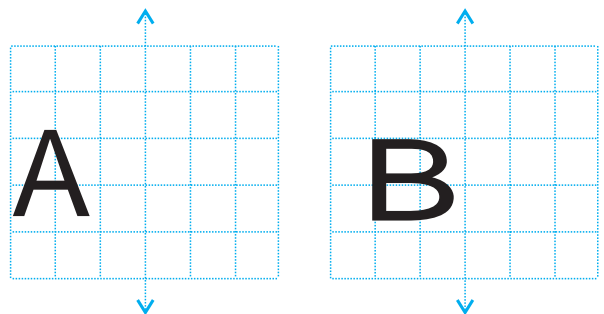


2. Copy the following drawing on squared paper. Complete each one of them such that the resulting figure has two dotted lines as two lines of symmetry.



How did you go about completing the picture?

3. In each figure alongside, a letter of the alphabet is shown along with a vertical line. Take the mirror image of the letter in the given line. Find which letters look the same after reflection (i.e. which letters look the same in the image) and which do not. Can you guess why?



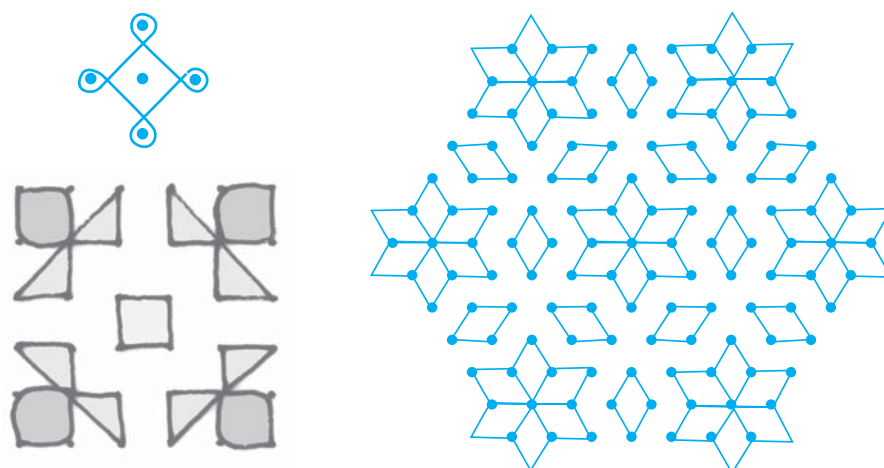
Try for **O E M N P H L T S V X**



Rangoli patterns

Kolams and Rangoli are popular in our country. A few samples are given here. Note the use of symmetry in them. Collect as many patterns as possible of these and prepare an album.

Try and locate symmetric portions of these patterns alongwith the lines of symmetry.



What have we discussed?

1. A figure has *line symmetry* if a line can be drawn dividing the figure into two identical parts. The line is called a *line of symmetry*.
2. A figure may have no line of symmetry, only one line of symmetry, two lines of symmetry or multiple lines of symmetry. Here are some examples.

<i>Number of lines of symmetry</i>	<i>Example</i>
No line of symmetry	A scalene triangle
Only one line of symmetry	An isosceles triangle
Two lines of symmetry	A rectangle
Three lines of symmetry	An equilateral triangle

3. The line symmetry is closely related to mirror reflection. When dealing with mirror reflection, we have to take into account the left ↔ right changes in orientation.

Symmetry has plenty of applications in everyday life as in art, architecture, textile technology, design creations, geometrical reasoning, Kolams, Rangoli etc.